Multi-Million Tons Freshwater TRANSPORTER with a Side of Calculus

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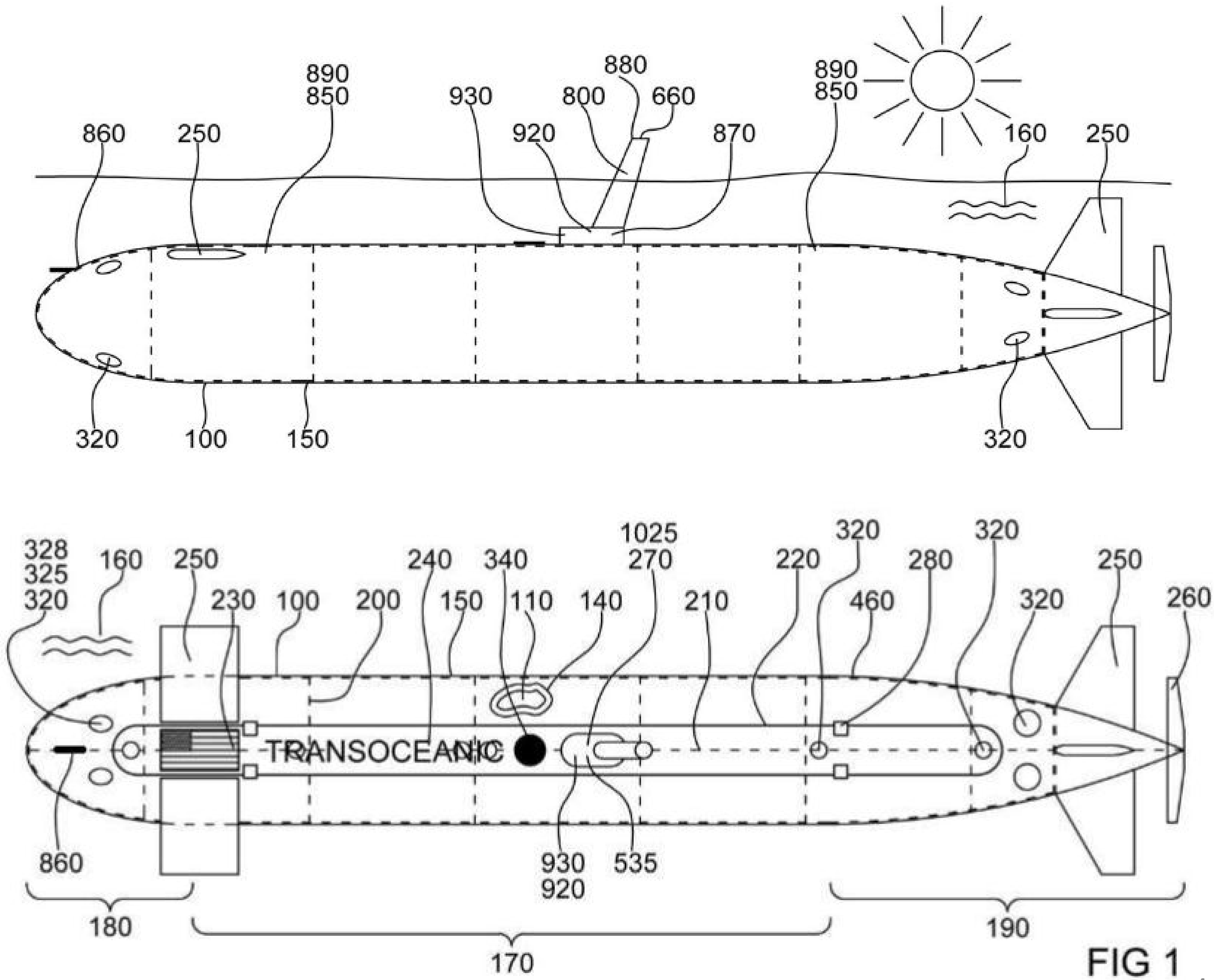
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FIG 1

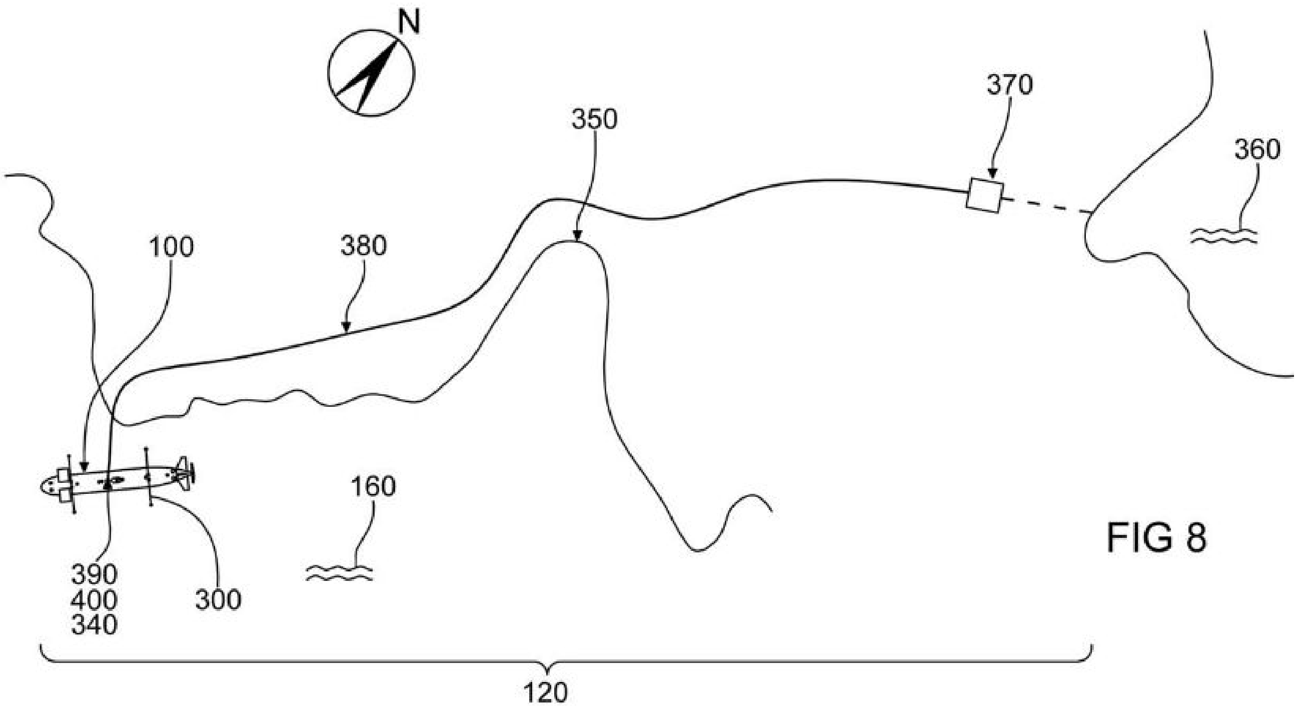

FIG 8

The **submersible transporters(100)** transport **freshwater(360)** from a **supply station(120)** …

…to a **delivery station(130)**

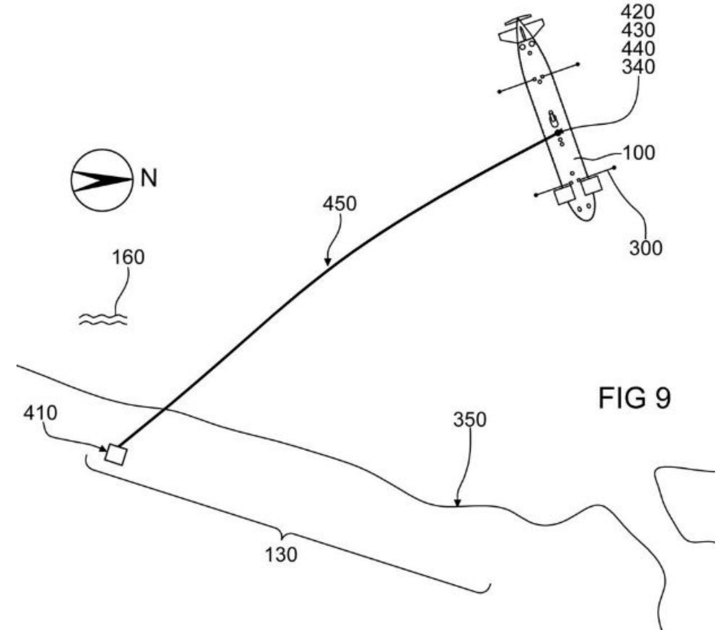

FIG 9

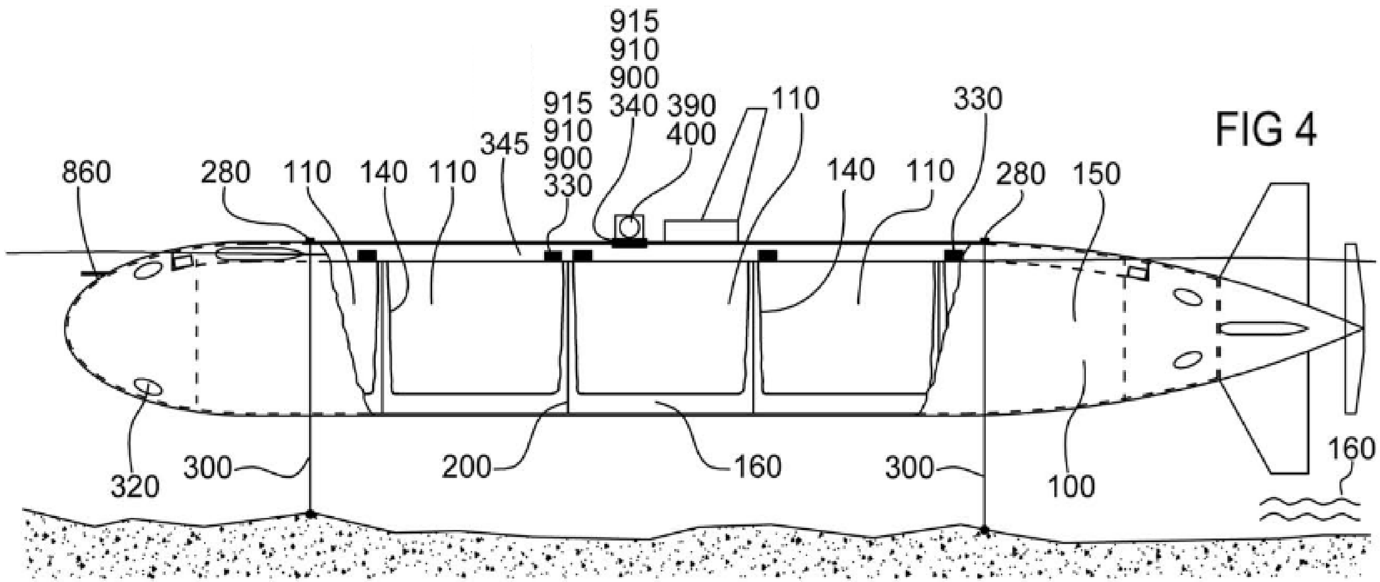
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FIG 4

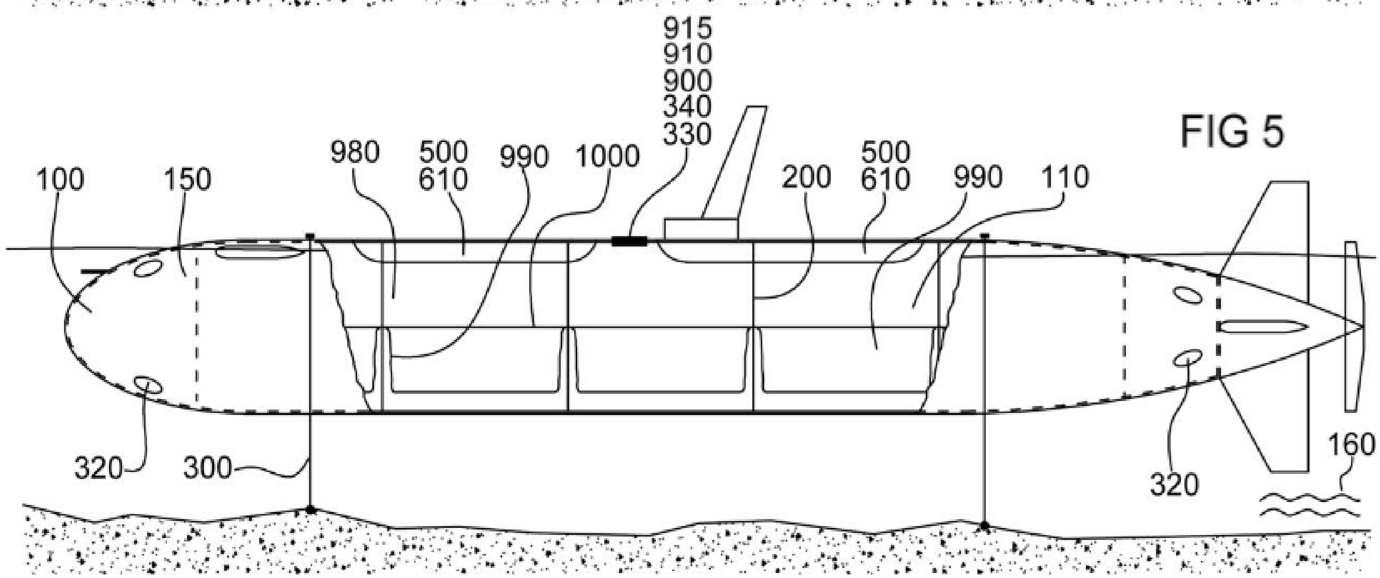
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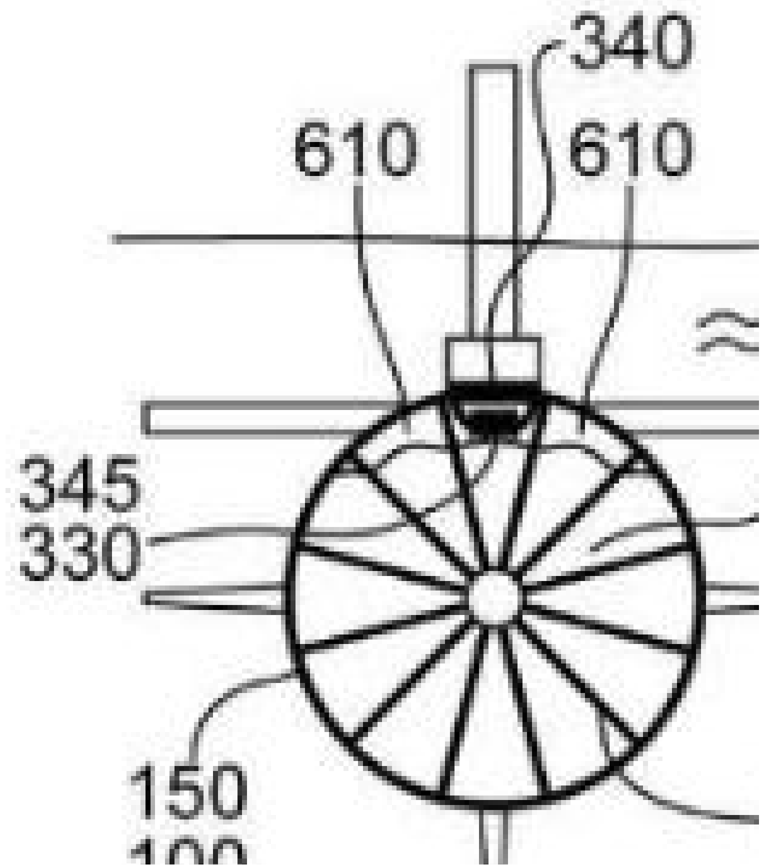
FIG 5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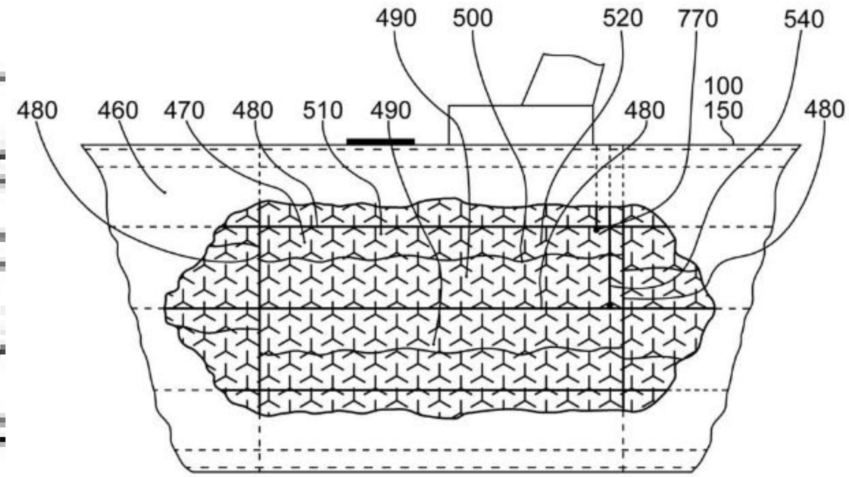
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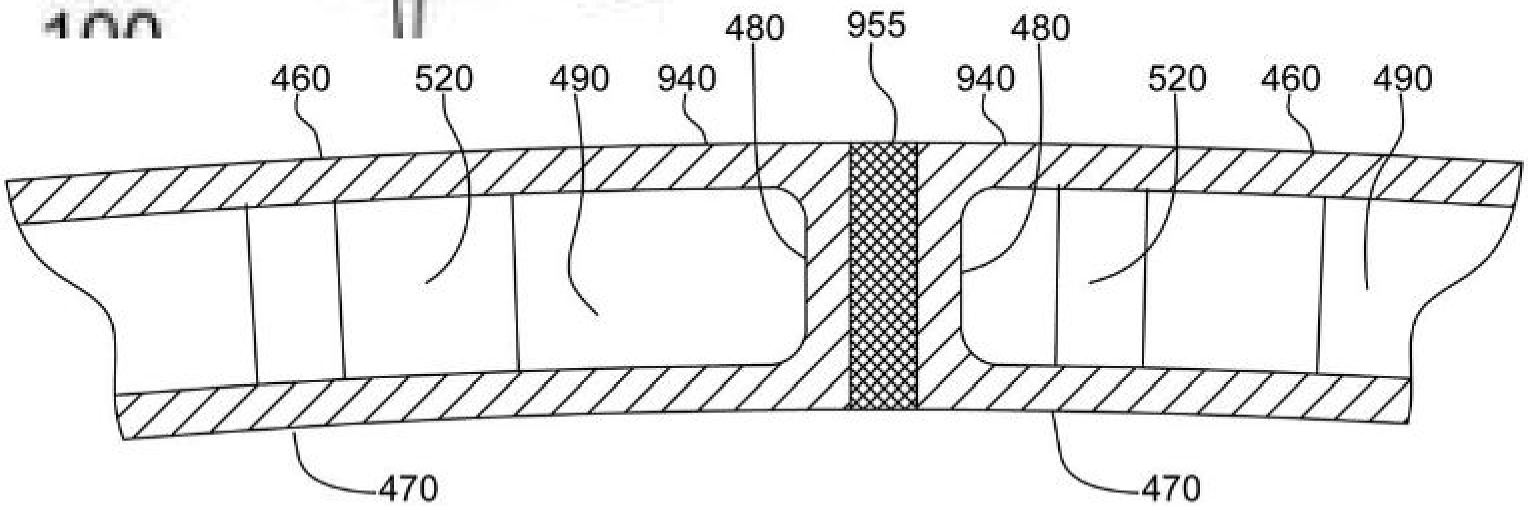
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1. An ultra large marine submersible transportation system for bulk liquids consisting of:

- **submersible transporters(100)** built to transport **bulk liquids(110)** across the oceans and seas
- from at least one specifically-built **supply station(120)**
- to at least one specifically-built **delivery station(130)**

2. The **submersible transporter(100)** typically have:

- lengths of 700 meters to 2400 meters (2300 to 8000 ft);
- diameters of 80 meters to 400 meters (270 to 1300 ft), and
- **liquids cargo capacity** from 3 million to over 120 million metric tons or cubic meters (2500 to 100,000 acre-feet); 10 to 400 times larger than Supertankers

3. The **submersible transporters(100) form A NEW CLASS OF SHIPS** provided with:

- some very large impervious **collapsible bladders(140)** enclosed in
- a reinforced concrete **submersible hull(150)** that is built with
  - a concrete **outer hull(460)** and
  - a concrete **inner hull(470)** joined by
  - some **separating partitions(480),**
  - together forming a multitude of separate impervious **ballast chambers(490)** that are ballasted independently of each other by partially and controllably filling them with some **ballast water(500)**

About the huge reinforced concrete ballast-chambered submersible hull(150):

- **It is built with the required ballast mass and variable buoyancy so it can ballast the submersible transporter(100) when full of freshwater; it can hold the submersible transporter(100) submerged or emerged at the sea surface;**

- **When submerged, it stays away from the swell and it comfortably holds its shape;**

- **It is hydrodynamic and creates a low induces wave resistance; it cruises economically and safely;**

- **It offers enough rigidity to holds the large inside collapsible bladders(140) sheltered from the open sea impact and the induced deformations**

- **It allows relatively simple operation modes with no-ballast water exchange;**

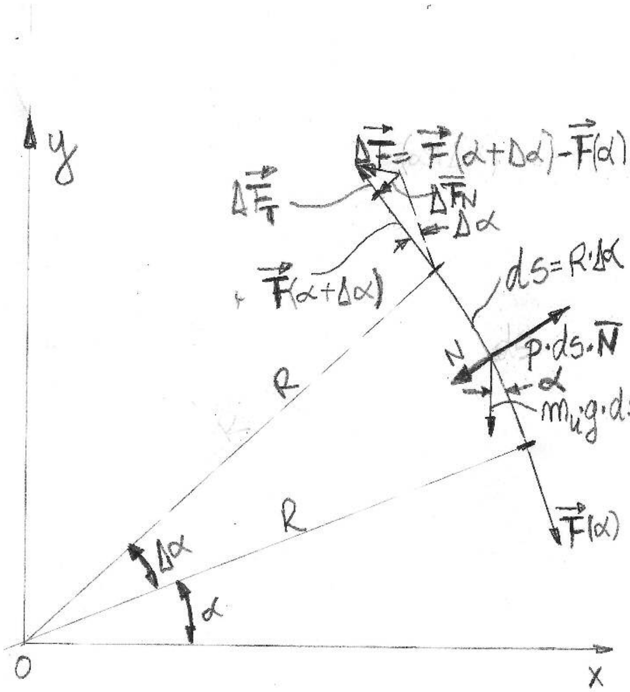
- **It is corrosion resistant and can economically be built as a ultra-large structure;**

- **It is a thin-shelled efficient structure that, being built with a round transversal section, is subject to minimal axial bending momentums at equilibrium.**

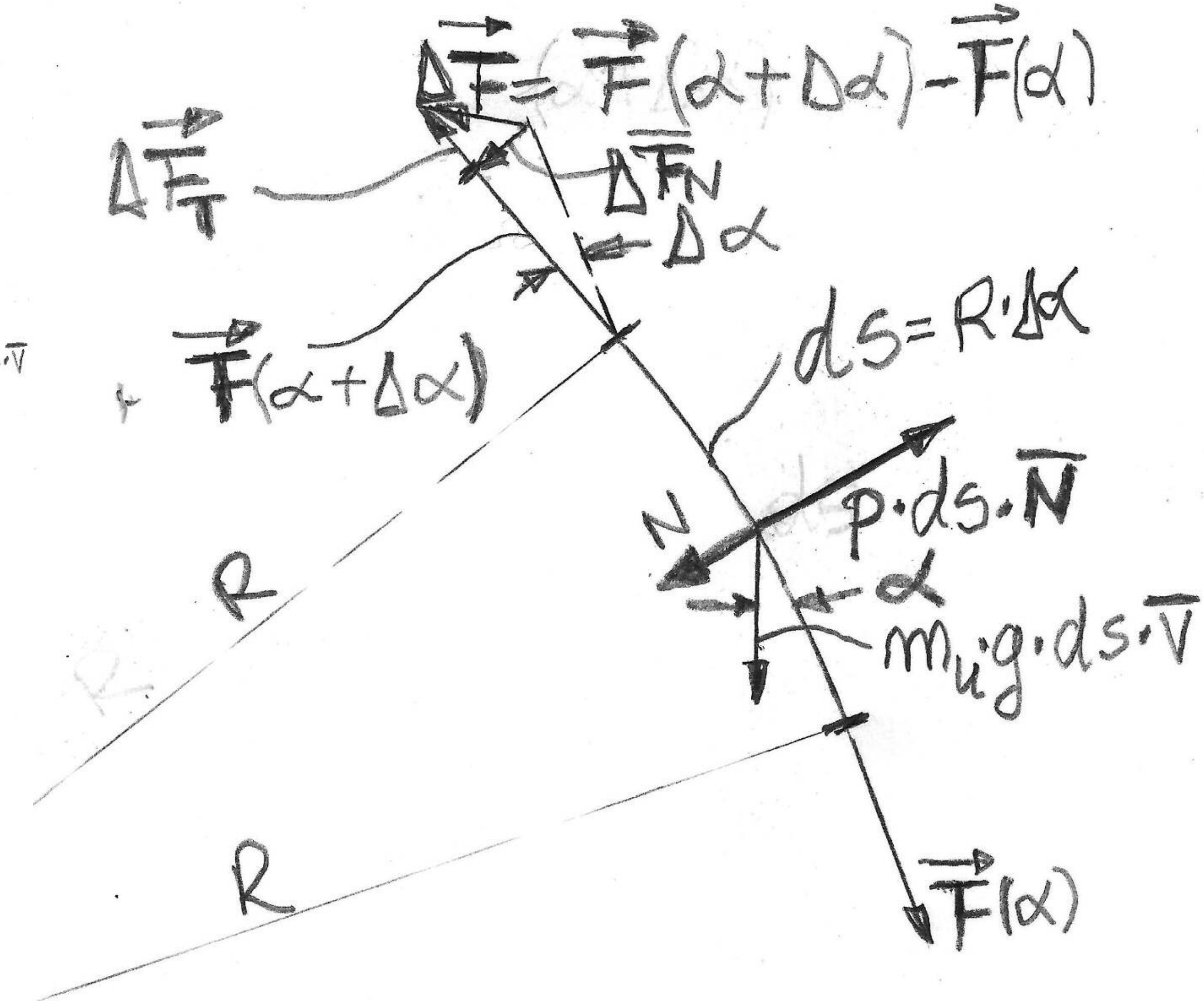
2. The promised side of calculus – first part

We shall verify the a circular uniformly ballasted transversal section through the thin-shelled hull of the transporter, in an equilibrium state, will present only tangent tension and no axial bending.
After writing the force equations, it is relatively simple to compute some of the parameters, including:

- The maximum tangential stress in the cylindrical part of the transporter;
- The ballasting requirements of the transporter.



y
x
0
$\Delta\vec{F} = \vec{F}(\alpha+\Delta\alpha) - \vec{F}(\alpha)$
$\Delta\vec{F}_T$
$\Delta\vec{F}_N$
$\Delta\alpha$
$\vec{F}(\alpha+\Delta\alpha)$
$ds = R\cdot\Delta\alpha$
$p\cdot ds\cdot\vec{N}$
N
α
$m_u\cdot g\cdot ds\cdot\vec{V}$
R
R
$\vec{F}(\alpha)$
$\Delta\alpha$
α



$\Delta\vec{F} = \vec{F}(\alpha+\Delta\alpha) - \vec{F}(\alpha)$
$\Delta\vec{F}_T$
$\Delta\vec{F}_N$
$\Delta\alpha$
$\vec{F}(\alpha+\Delta\alpha)$
$ds = R\cdot\Delta\alpha$
$p\cdot ds\cdot\vec{N}$
N
α
$m_u\cdot g\cdot ds\cdot\vec{V}$
R
R
$\vec{F}(\alpha)$

Computation of tension and pressure for the circular membrane

We consider the following notations:

- $p_o^I =$ interior pressure at the origin O (kg/m)
- $p_o^E =$ exterior pressure at the origin O (kg/m)
- $p_o^I - p_o^E$
- $\rho_I =$ the density of the interior liquid (kg/m^2)
- $\rho_E =$ the density of the exterior liquid (kg/m^2)
- $g =$ gravitational constant (m/s^2)
- $m_u =$ constant ballast on the membrane the membrane (kg/m)
- R radius of the membrane(m)
- $L = 2\pi R$ is the length of the membrane (m)
- $p^I =$ interior pressure (N/m)
- $p^E =$ exterior presssure (N/m)
- $p = p^I - p^E$
- $s = R\alpha =$ length of arc from the Ox axis
- $\vec{F} =$ tension force in the membrane(N)
- $F = |\vec{F}|$ module of tension force $\vec{F}$

Let us place our system of coordinates at the center of the circular membrane as in the figure. Let $\vec{F}$ denote the tension in the membrane and let $\Delta\vec{F}$ denote the force acting onto an infinitesimal element of the membrane segment

$$\Delta s = R\Delta\alpha$$

(IEM for short).

$\Delta\vec{F}$ has a normal component $\Delta\vec{F}_N$ and a tangent component $\Delta\vec{F}_T$ to the IEM. The vector $\vec{F}$ has no normal component.

Radial equilibrium implies

$$(1)\quad |\Delta\vec{F}_N| - pR\Delta\alpha + m_u gR\Delta\alpha \sin\alpha = 0$$

and since

$$(2)\quad |\Delta\vec{F}_N| = |\vec{F}|\Delta\alpha = F\Delta\alpha$$

it follows that

$$(3)\quad F = pR - m_u gR\sin\alpha$$

Tangential equilibrium is satisfied when

$$(4)\quad \Delta F - m_u g R \Delta\alpha \cos\alpha = 0$$

In the above equation dividing by $\Delta\alpha$ and passing to the limit we obtain

$$(5)\quad \frac{dF}{d\alpha} = m_u R g \cos\alpha$$

Next integrating (5) we obtain

$$(6)\quad F = m_u g R(\sin\alpha + c_F)$$

and to minimize F and avoid buckling ($F \geq 0$) we can take

$$c_F = 1$$

and therefore

$$(7)\quad F = m_u g R(\sin\alpha + 1)$$

Considering

$$(8)\quad m_u = (\rho_E - \rho_I)\frac{R}{2}$$

for Archimedean equilibrium we get

$$(9)\quad F = (\frac{\rho_E - \rho_I}{2}) g R^2 (\sin\alpha + 1)$$

and therefore the maximum F(at $\alpha = \frac{\pi}{2}$) will be given by

$$(10)\quad F = (\rho_E - \rho_I) g R^2$$

The pressure for the case of $c_F = 1$ is

$$(11)\quad \frac{F}{R} + m_u g \sin\alpha = 2 m_u g(\sin\alpha + \frac{1}{2})$$

and also

$$(12)\quad p < 0 \text{ for } \alpha < -\frac{\pi}{6}.$$

Achieving $p < 0$ is problematic in some engineering configurations. That is why we shall analyze the case $p > 0$ anywhere on our membrane.

From (3) and (6) we get

$$p = \frac{F}{R} + m_u g \sin\alpha$$

and

$$p = m_u g(2\sin\alpha + c_F).$$

For $p \geq 0$ for any value of α it results that

$$(13) \quad c_F \geq 2$$

and for $c_F = 2$ we get

$$(14) \quad F = m_u g R(\sin\alpha + 2),$$

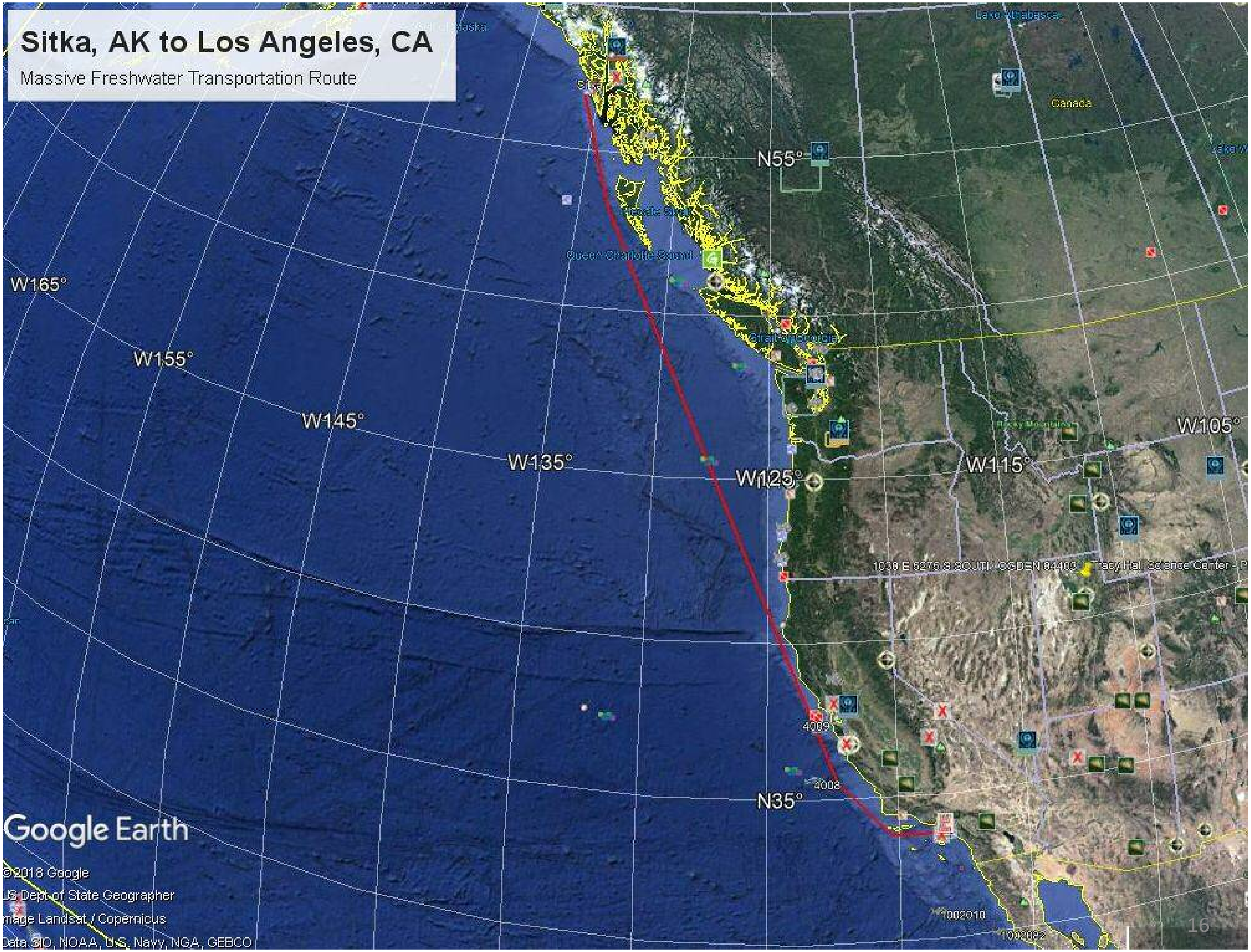
and for $\alpha = \frac{\pi}{2}$

$$(15) \quad F_{max} = \frac{3}{2}(\rho_E - \rho_I) g R^2$$

3. Description of a demonstration project and its economics

Sitka, AK to Los Angeles, CA

Massive Freshwater Transportation Route



SUBMERSIBLE TRANSPORTER GEOMETRY	
Radius, m	45
Radius, ft	148
L/(2R) RATIO	7
Equivalent submersible length, m	630
Submersible Section, square meters	6362
Submersible surface (considered closed by hemispheres)	203575
Volume (considered cylindrical for equivalent length), cubic meters	4,007,883
volume, acre-ft	3249.249

BALLASTING	
Relative density of concrete	2.4
Ballasting required (on cylindrical part), kg/square meters	630
Thickness of concrete wall, m	0.459
Volume of concrete required, cubic meters	93478
Volume of concrete required, cubic yards	122265
Weight of concrete structure, tons	224348
Chamber thickness, m	0.63
Total hull thickeness, m	1.09

STRESSES	
Max tangent force, Newton/linear meter	834,341
Max tangent force, metric ton force/linear meter	85.050

HYDRODYNAMICS	
cx (drag coefficient)	0.08
Speed, cruising, v, m/s	3.5
Speed cruising km/h	12.6
Drag force, Newton	3204525
Drag force, metric ton force	326.659
Power required for cruising (w)	11215839
Power for cruising, Mw	11.216

POWER AND ENGINE	
Propulsor efficiency	0.7
Required Engine power, Mw	16.023
Engine power reserve, %	20%
Total Engine Installed Power, Mw	19.227

FIXED COSTS	
SUBMERSIBLE COST	
Unit Concrete price, $/cubic meter	70.00
Unit cost of armature, $/cubic meter	60.00
Unit cost of work, $/cubic meter	120.00
Total unit price of concrete, $/cubic meter	250.00
Cost of concrete for submersible, $	$ 23,369,583
PROPULSOR COSTS	
Cost of engine and propeller $/Mw	500,000
Cost of propulsion	$ 9,613,576
INSTRUMENTATION, COMMAND AND CONTROL (ICC)	
Cost of ICC	$ 8,000,000
Total cost of submersible	**$ 40,983,159.17**

VARIABLE COSTS (FUEL)	
FUEL COST PER HOUR	
Fuel Consumption, kg/kWh	0.165
Fuel cost (LNG), $/kg	0.2207
Cost of fuel per hour (at required power) $/hour	584

TRANSPORTATION PARAMETERS	
Supply station	Sitka, AK
Delivery station	Los Angeles, CA
Distance, one way, km	3100
Stationing, days at each station	3
Cruise time, round trip, hours	492.06
Cruise time, round trip, days	20.50
Total travel time, hours	636
Total travel time, days	26.50
Fuel cost per trip (when stationary, at half consumption per hour), $	329141
Amortization of submersible (25 year life) $/hour	187.14
Cost of submersible per trip	119031
Net cost per trip (submersible+fuel)	448172
Net transport cost $/acre-ft	**137.93**
G&A + profit	30%
Brut cost per trip, $	582624
Brut transport cost $/acre-ft	**179.31**

TRANSPORTATION SYSTEM CONFIGURATION AND COST	
Available water, acre-ft per year	320,000
Transporter capabillty, trips/year	13.772
Transporter capability, acre-ft/year	44749
Required number of transporters	8
Cost of transporters	$327,865,273
Cost of two stations (each equal to one transporter cost)	$81,966,318
Total cost of transportation system	**$409,831,592**

PARAMETERS	
Investment cost $/(acre-ft/year)	1281
Pressure drop equivalent, meters	505
Water flow at stations for filling/emptying, cu m/sec	15.46
Transportation system capability (acre-ft/year)	357,995

4. The promised side of calculus – final part

In what follows we will calculate the exact shape of a membrane containing a liquid of density ρ_I that is immeresed in a liquid of density ρ_E and in dynamic equilibrium. The membrane can be imagined as being the crossection of a cylindrical container immersed in salt water and containing fresh water.

Assumptions, notations and figure. The thickness of the membrane is negligible. The membrane cannot be stretched nor compressed in the tangent direction.

We will consider an arc length parametrization of the membrane

$$x = x(s), y = y(s) \tag{1}$$

where the arc-length parameter is

$$0 \leq s \leq L.$$

The unit length tangent vector is

$$\vec{T} =< x'(s), y'(s) >$$

The tangent vector is oriented counterclockwise and we obviously have

$$|\vec{T}| = (x')^2 + (y')^2 = 1 \tag{2}$$

The unit length vector normal to $\vec{T}$ and pointing towards the interior of the membrane is

$$\vec{N} =< y', -x' > \tag{3}$$

In the figure Δs is assumed to be an infinitesimal element of the membrane. The forces acting upon Δs are

$$\vec{G} =< 0, m_u g \Delta s >, \tag{4}$$

the hydrostatic force

$$\vec{H} = -p\Delta s \cdot \vec{N} \tag{5}$$

where $p = p^I - p^E$ at Δs. The third force acting on Δs is the tension in the membrane denoted $\frac{d\vec{F}}{ds}$ where

$$\vec{F} = f(s) \cdot \vec{T} \tag{6}$$

The IEM will be in equilibrium if

$$\vec{G} + \vec{H} + \frac{d\vec{F}}{ds}\Delta s = 0 \tag{7}$$

Breaking it down component-wise(in the x and y directions) we get

$$\begin{cases} -py' + f'y' + fy'' = 0 \\ m_u g + px' + f'y' + fy'' = 0 \end{cases} \tag{8}$$

We also need to take into account that

$$\frac{d\vec{T}}{ds} = k(s) \cdot \vec{N} \tag{9}$$

where $k(s)$ denotes the curvature of the membrane(**Not assumed to be constant!**). Component-wise (9) gives

$$\begin{cases} x'' = k(s)y' \\ y'' = -k(s)x' \end{cases} \tag{10}$$

Manipulating the system (8) algebraically and differentially we were able to prove that

$$k' = 0,$$

hence the curvature of the membrane is constant.

References:

https://transoceanic.us/
Google Earth